1. (a) $y' = 3\sin 2x + 6x\cos 2x$ M1

$y'' = 12\cos 2x - 12x\sin 2x$ A1

Substituting $12\cos 2x - 12x\sin 2x + 12x\sin 2 = k\cos 2x$ M1

$k = 12$ A1 4

(b) General solution is $y = A\cos 2x + B\sin 2x + 3x\sin 2x$ B1

$(0, 2) \Rightarrow A = 2$ B1

$\left(\frac{\pi}{4}, \frac{\pi}{2}\right) \Rightarrow \frac{\pi}{2} = B + \frac{3\pi}{4} \Rightarrow B = -\frac{\pi}{4}$ M1

$y = 2\cos 2x - \frac{\pi}{4}\sin 2x + 3x\sin 2x$ Needs $y = \ldots$ A1 4

[8]

2. (a) $(2r+1)^3 = 8r^3 + 12r^2 + 6r + 1$

$(2r-1)^3 = 8r^3 - 12r^2 + 6r - 1$

$(2r+1)^3 - (2r-1)^3 = 24r^2 + 2$ ($A = 24$, $B = 2$) M1 A1 2

Accept $r = 0 \Rightarrow B = 2$ and $r = 1 \Rightarrow A + B = 26 \Rightarrow A = 24$

M1 for both

(b)

$3^3 - 1^3 = 24 \times 1^2 + 2$

$5^3 - 3^3 = 24 \times 2^2 + 2$

M

$(2n+1)^3 - (2n-1)^3 = 24 \times n^2 + 2$

$(2n+1)^3 - 1^3 = 24\sum_{r=1}^{n} r^2 + \underline{2n}$ ft their B M1 A1 A1ft

$\sum_{r=1}^{n} r^2 = \frac{8n^3 + 12n^2 + 4n}{24}$ M1

$= \frac{1}{6}n(2n^2 + 3n + 1) = \frac{1}{6}n(n+1)(2n+1)$ cso A1 5

(c) $\sum_{r=1}^{40}(3r-1)^2=\sum_{r=1}^{40}(9r^2-6r+1)$ M1

$=9\times\frac{1}{6}\times40\times41\times81-6\times\frac{1}{2}\times40\times41+40$ M1

= 194380 A1 3

[10]

3. (a) $2x^2 + x - 6 = 6 - 3x$ M1

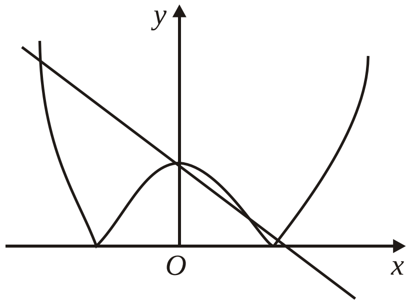
Leading to $x^2 + 2x - 6 = 0$

$(x + 1)^2 = 7 \Rightarrow x = -1 \pm \sqrt{7}$ surds required M1 A1

$-2x^2 - x + 6 = 6 - 3x$ M1

Leading to $2x^2 - 2x = 0, \Rightarrow x = 0, 1$ A1 A1 6

(b) Accept if parts (a) and (b) done in reverse order



Curved shape B1

Line B1

At least 3 intersections B1 3

(c) Using all 4 CVs and getting all into inequalities M1

$x > \sqrt{7} - 1$, $x < -\sqrt{7} - 1$ both A1ft

ft their greatest positive and their least negative CVs

$0 < x < 1$ A1 3

[12]

4. (a) $\int \frac{2}{120-t}\,dt = -2\ln(120-t)$ B1

$e^{-2\ln(120-t)} = (120-t)^{-2}$ M1 A1

$$\frac{1}{(120-t)^2}\frac{ds}{dt} + \frac{2S}{(120-t)^3} = \frac{1}{4(120-t)^2}$$

$\frac{d}{dt}\left(\frac{S}{(120-t)^2}\right) = \frac{1}{4(120-t)^2}$ or integral equivalent M1

$\frac{S}{(120-t)^2} = \frac{1}{4(120-t)}(+C)$ M1 A1

$(0, 6) \Rightarrow 6 = 30 + 120^2 C \Rightarrow C = -\frac{1}{600}$ M1

$S = \frac{120-t}{4} - \frac{(120-t)^2}{600}$ accept C = awrt –0.0017 A1 8

(b) $\frac{dS}{dt} = -\frac{1}{4} + \frac{2(120-t)}{600}$ M1

$\frac{dS}{dt} = 0 \Rightarrow t = 45$ M1 A1

substituting $S = 9\frac{3}{8}$ (kg) A1 4

[12]

Alternative forms for S are

$$S = 6 + \frac{3t}{20} - \frac{t^2}{600} = \frac{(t+30)(120-t)}{600}$$

$$= \frac{3600 + 90t - t^2}{600} = \frac{5625 - (t-45)^2}{600}$$

Alternative for part (b)

S can be found without finding t

Using $\frac{dS}{dt} = 0$ in the original differential equation $\frac{2S}{120 - t} = \frac{1}{4}$ M1

Substituting for t into the answer to part (a)

$S = 2S - \frac{64S^2}{600}$ M1 A1

Solving to $S = 9\frac{3}{8}$ (kg) A1 4

5. (a) $f(x) = \cos 2x$, $f(\frac{\pi}{4}) = 0$

$f'(x) = -2 \sin 2x$, $f'(\frac{\pi}{4}) = -2$ M1

$f''(x) = -4 \cos 2x$, $f''(\frac{\pi}{4}) = 0$

$f'''(x) = 8 \sin 2x$, $f'''(\frac{\pi}{4}) = 8$ A1

$f^{(iv)}(x) = 16 \cos 2x$, $f^{(iv)}(\frac{\pi}{4}) = 0$

$f^{(v)}(x) = 32 \sin 2x$, $f^{(v)}(\frac{\pi}{4}) = -32$ A1

$\cos 2x = f(\frac{\pi}{4}) + f'(\frac{\pi}{4})(x - \frac{\pi}{4}) + \frac{f''(\frac{\pi}{4})}{2}(x - \frac{\pi}{4})^2 + \frac{f'(\frac{\pi}{4})}{3!}(x - \frac{\pi}{4})^3 + \ldots$ M1

Three terms are sufficient to establish method

$\cos 2x = -2(x - \frac{\pi}{4}) + \frac{4}{3}(x - \frac{\pi}{4})^3 - \frac{4}{15}(x - \frac{\pi}{4})^5 + ..$ A1 5

(b) Substitute $x = 1$ $(1 - \frac{\pi}{4} \approx 0.21460)$ B1

$\cos 2 = -2(x - \frac{\pi}{4}) + \frac{4}{3}(x - \frac{\pi}{4})^3 - \frac{4}{15}(x - \frac{\pi}{4})^5 + ..$

≈ -0.416147 cao M1 A1 3

[8]

6. (a) In this solution $\cos\theta$ = c and $\sin\theta$ = s

$\cos 5\theta + i\sin 5\theta = (c + is)^5$ — M1

$(= c^5 + 5c^4\, is + 10c^3 (is)^2 + 10c^2 (is)^3 + 5c\,(is)^4 + (is)^5)$

$\sin 5\theta = 5c^4 s - 10c^2 s^3 + s^5$ — M1 A1

$= 5c^4 s - 10c^2(1 - c^2)s + (1 - c^2)^2 s$ $s^2 = 1 - c^2$ — M1

$= s\,(16c^4 - 12c^2 + 1)$ — A1 5

(b) $\sin\theta(16\cos^4\theta - 12\cos^2\theta + 1) + 2\cos^2\theta\sin\theta = 0$ — M1

$\sin\theta = 0 \Rightarrow \theta = 0$ — B1

$16c^4 - 10c^2 + 1 = (8c^2 - 1)(2c^2 - 1) = 0$ — M1

$c = \pm\frac{1}{2\sqrt{2}},\ c = \pm\frac{1}{\sqrt{2}}$ any two — A1

$\theta \approx 1.21, 1.93;\ \theta = \frac{\pi}{4}, \frac{3\pi}{4}$ any two — A1

all four — A1 6

accept awrt 0.79, 1 21,1.93,2.36

Ignore any solutions out of range.

[11]

7. (a) $\left(\frac{dx}{dt}\right)_0 = 0.4 \approx \frac{x_{0.1} - 0}{0.1} \Rightarrow x_{0.1} \approx 0.04$ — B1

$\left(\frac{d^2x}{dt^2}\right)_{0.1} = 3\sin x_{0.1} \approx \frac{x_{0.2} - 2x_{0.1} + 0}{0.01}$ — M1

Must have their $x_{0.1}$

$x_{0.2} \approx 0.0788$ awrt — A1

$\left(\frac{d^2x}{dt^2}\right)_{02} = 3\sin x_{0.2} \approx \frac{x_{0.3} - 2x_{0.2} + x_{0.1}}{0.01}$ — M1

Must have their $x_{0.1}$, $x_{0.2}$

$x_{0.3} \approx 0.115$ awrt — A1 5

(b) $f''(t) = -3\sin x$, $f''(0) = 0$

$f'''(t) = -3\cos\ x\dfrac{dx}{dt}$, $f'''(0) = -3\times0.4=-1.2$ M1 A1

$f(t) = f(0) + f'(0) + \dfrac{t^2}{2}f''(0)+\dfrac{t^3}{3!}f'''(0)+....$

$=0.4t - 0.2t^3$ M1 A1 4

(c) Substituting $t = 0.3$ into their answer to (b) and evaluating M1
$f(0.3) \approx 0.1146$ cao A1 2
[11]

8. (a) Let $z = x + iy$

$(x - 6)^2 + (y + 3)^2 = 9[(x + 2)^2 + (y - 1)^2]$ M1

Leading to $8x^2 + 8y^2 + 48x - 24y = 0$ M1 A1

This is a circle; the coefficients of x^2 and y^2 are the same and there is no xy term.

Allow equivalent arguments and ft their f, (x, y) if appropriate. A1ft

$(x^2 + 6x + y^2 - 3y = 0)$

Leading to $(x + 3)^2 + (y - \frac{3}{2})^2 = \frac{45}{4}$ M1

Centre: $(-3, \frac{3}{2})$ A1

Radius: $\frac{3}{2}\sqrt{5}$ or equivalent Al 7

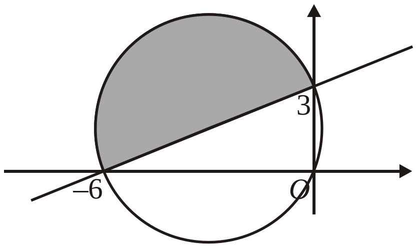
Alternative
Accept the following argument:-
The locus of P is a Circle of Apollonius, which is a circle with diameter XY, where the points X and Y cut (6, –3) and (–2, 1) internally and externally in the ratio 3 : 1. *M1 A1*

X: (0, 0) Y: (–6, 3) *M1 A1*

Centre: $(-3, \frac{3}{2})$ *M1 A1*

Radius: $\frac{3}{2}\sqrt{5}$ *or equivalent* *A1* *7*

(b)



Circle	B1	
centre in correct quadrant	B1 ft	
through origin	B1	
Line cuts -ve x and +ve y axes	B1	
intersects with circle on axes and all correct	B1	5

(c)

Shading inside circle	B1	
and above line with all correct	B1	2

Having 3 instead of 9 in first equation gains maximum of
M1M1A0A1ftM1A0A0 B1B1B0B1B0 B1B0 8/14

[14]